

# PLL is a phase-shift feedback

① Costas Receiver

② Squaring ~~Feedback~~ Loop

I-Channel

$$\frac{1}{2} A_c \cos \phi_a m(t)$$

In Phase Channel

Product Modulator

LPF

$$\cos(2\pi f_c t + \phi)$$

VCO

error signal  
dc-value

Phase Discriminator

$$A_c \cos(2\pi f_c t) m(t) + \phi_i$$

$$\sin(\phi_i - \phi_a)$$

90° Phase Shifter

$$\sin(2\pi f_c t + \phi)$$

Quadrature Channel

Product Modulator

LPF

$$\frac{1}{2} A_c m(t) \cdot \sin \phi$$

90° ~~phase shift~~ - Q-Channel

VCO → Voltage Control Oscillator

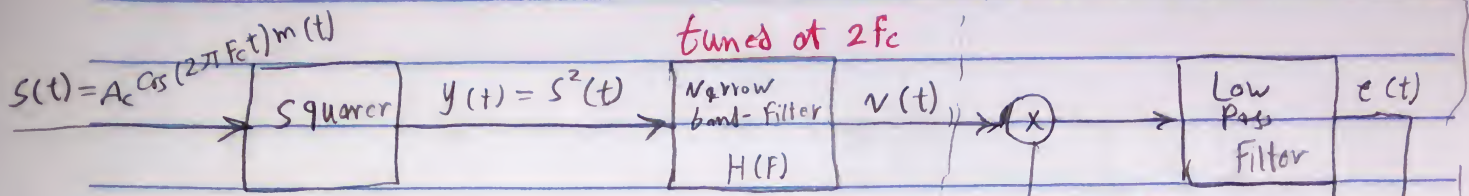
phase-tracking

feedback



## ② Squaring Loop

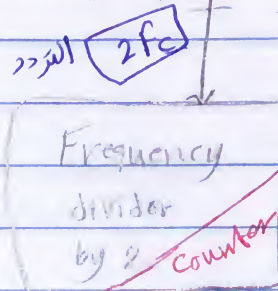
PLL



$$y(t) = A_c^2 m^2(t) \cos^2(2\pi F_c t)$$

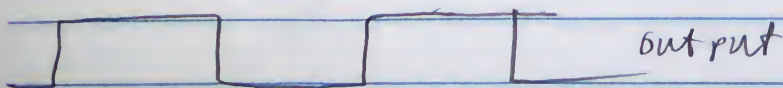
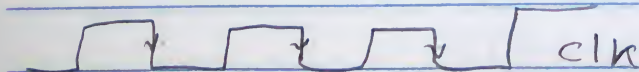
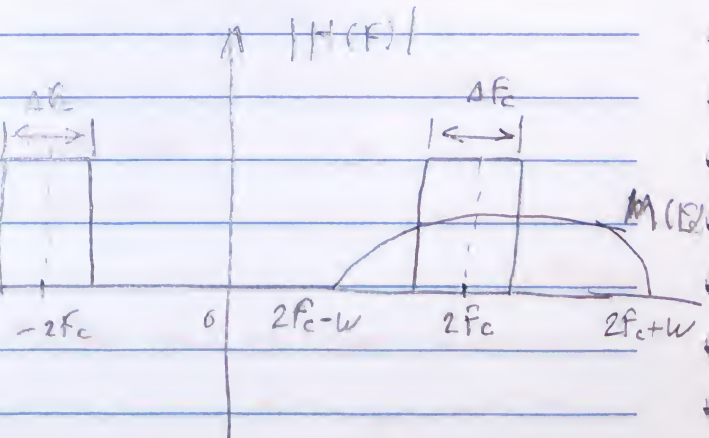
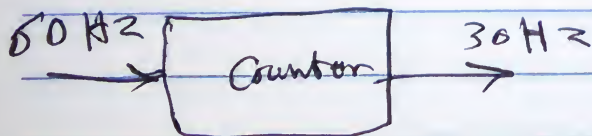
$$= \frac{1}{2} A_c^2 m^2(t) [1 + \cos(4\pi F_c t)]$$

$$V(t) = A_c E \Delta F \cos(4\pi F_c t)$$



$e(t)$  → difference between  
Frequency (phase  $v(t)$ )  
and Frequency and phase of VCO

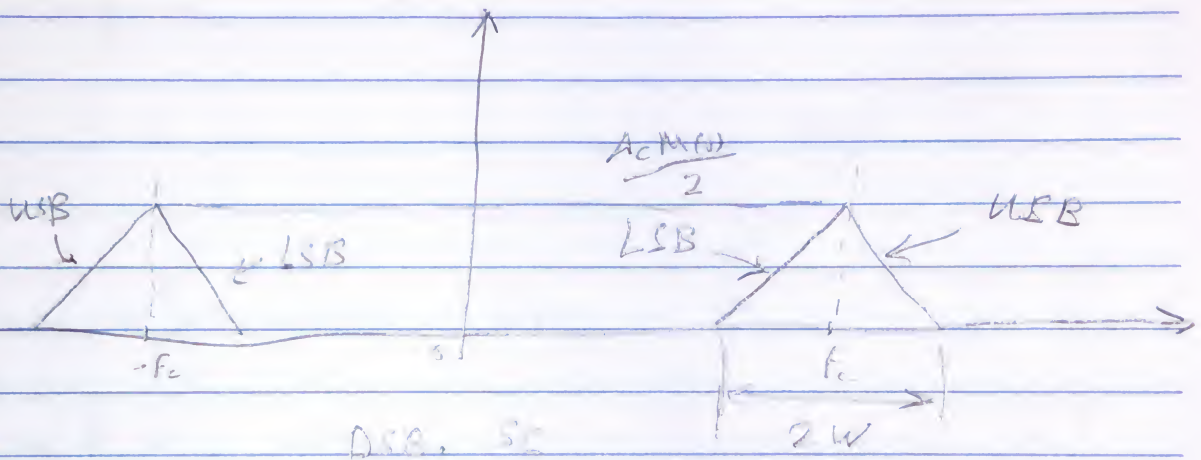
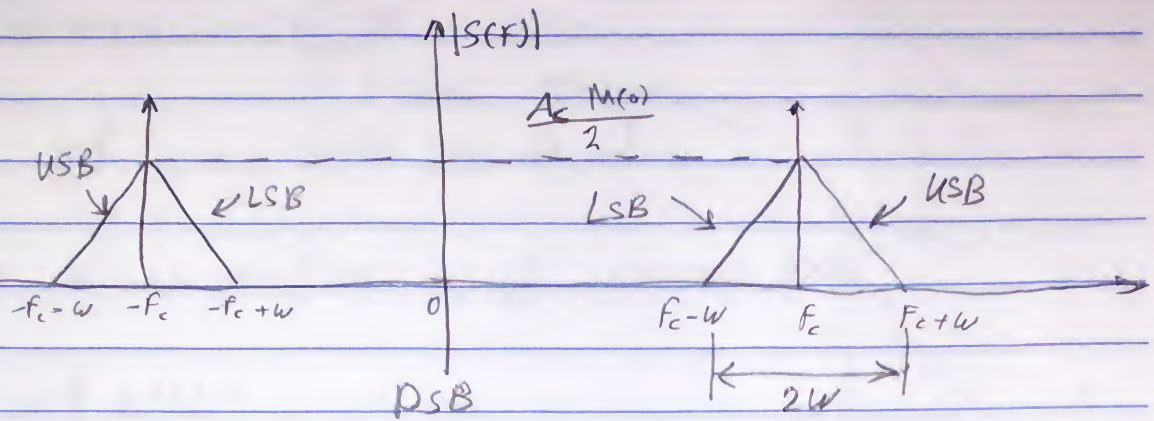
Carrier wave at  $F_c$  frequency



bit \* 0



# Single side band Modulation (SSB)



في حال Single sideband

\* يوتر في Band width

\* يوتر في Power

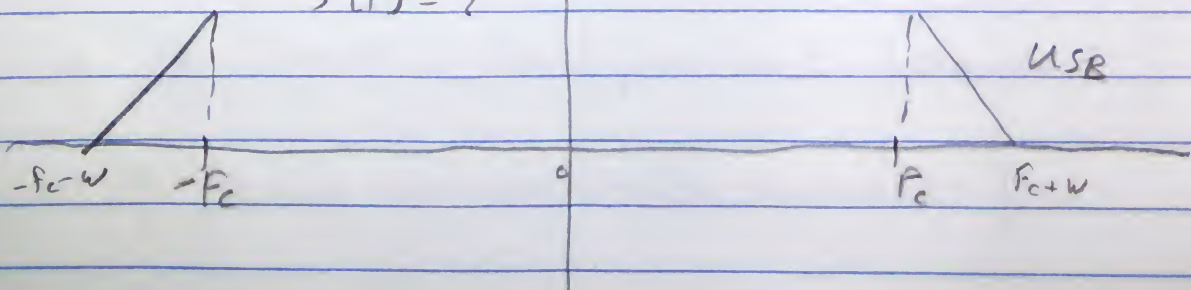
في حال Single sideband

\* يجب ان Perfect band pass filter

\* في حال Perfect band pass filter

$$S(t) = ?$$

$$S(f) = ?$$



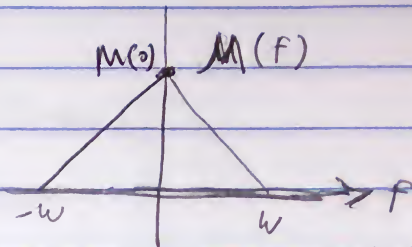


pre-envelope of  $m(t)$ :

$$m_+(t) = m(t) + j \hat{m}(t)$$

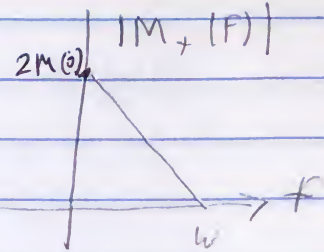
↓ Hilbert transform

$90^\circ$  phase shift (use of)



$$\hat{m}(t) = m(t) \text{ shifted by } \frac{\pi}{2} \text{ (Phase)}$$

$$M_+(f) = \begin{cases} 2M(f) & f > 0 \\ 0 & f < 0 \end{cases}$$

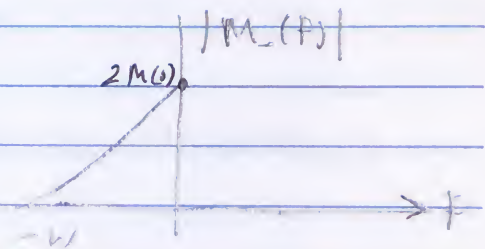


~~$M_-(f)$~~

$$m_-(t) = m_+^*(t) = m(t) - j \hat{m}(t)$$

$$M_-(f) = \begin{cases} 0 & f > 0 \\ 2M(f) & f < 0 \end{cases}$$

$$\hat{M}(f) = M(f) \cdot e^{j\frac{\pi}{2}}$$



To get single side band:

Upper Side Band

$$s(t) = \left[ m_+(t) e^{j2\pi f_c t} + m_-(t) e^{-j2\pi f_c t} \right] \frac{A_c}{4}$$

$$s(t) = \frac{A_c}{4} \left[ (m(t) + j \hat{m}(t)) (\cos(2\pi f_c t) + j \sin(2\pi f_c t)) \right]$$

$$+ \frac{A_c}{4} \left[ (m(t) - j \hat{m}(t)) (\cos(2\pi f_c t) - j \sin(2\pi f_c t)) \right]$$

$$= \frac{A_c}{4} \left[ 2m(t) \cos(2\pi f_c t) - 2\hat{m}(t) \sin(2\pi f_c t) \right]$$

$$= \frac{A_c}{2} \left[ m(t) \cos(2\pi f_c t) - \hat{m}(t) \sin(2\pi f_c t) \right]$$





Lower Side Band

$$S(t) = \frac{A_c}{2} [ m(t) \cos(2\pi f_c t) + \hat{m}(t) \sin(2\pi f_c t) ]$$

For single side band

$$S(t) = \frac{A_c}{2} [ m(t) \cos(2\pi f_c t) \mp \hat{m}(t) \sin(2\pi f_c t) ]$$

Ex:  $m(t) = A_m \cos(2\pi f_m t)$

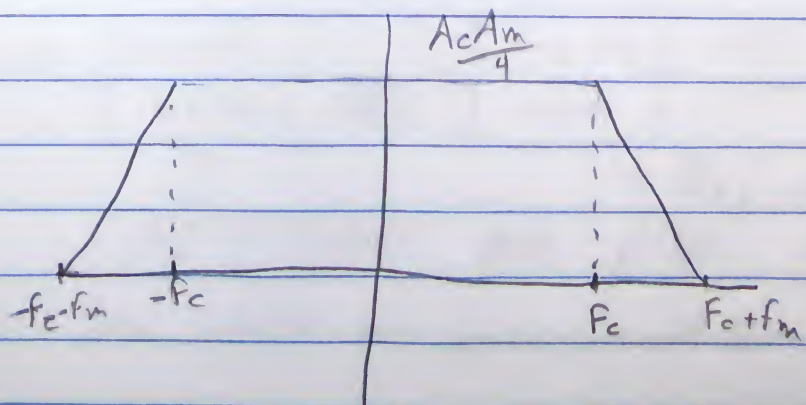
Amplitude  $A_m$

$$\hat{m}(t) = A_m \cos(2\pi f_m t + \frac{\pi}{2}) = A_m \sin(2\pi f_m t)$$

$$S(t) = \frac{A_c}{2} [ A_m \cos(2\pi f_m t) \cos(2\pi f_c t) - A_m \sin(2\pi f_m t) \sin(2\pi f_c t) ]$$

$$= \frac{A_c A_m}{2} \cos(2\pi (f_m + f_c) t)$$

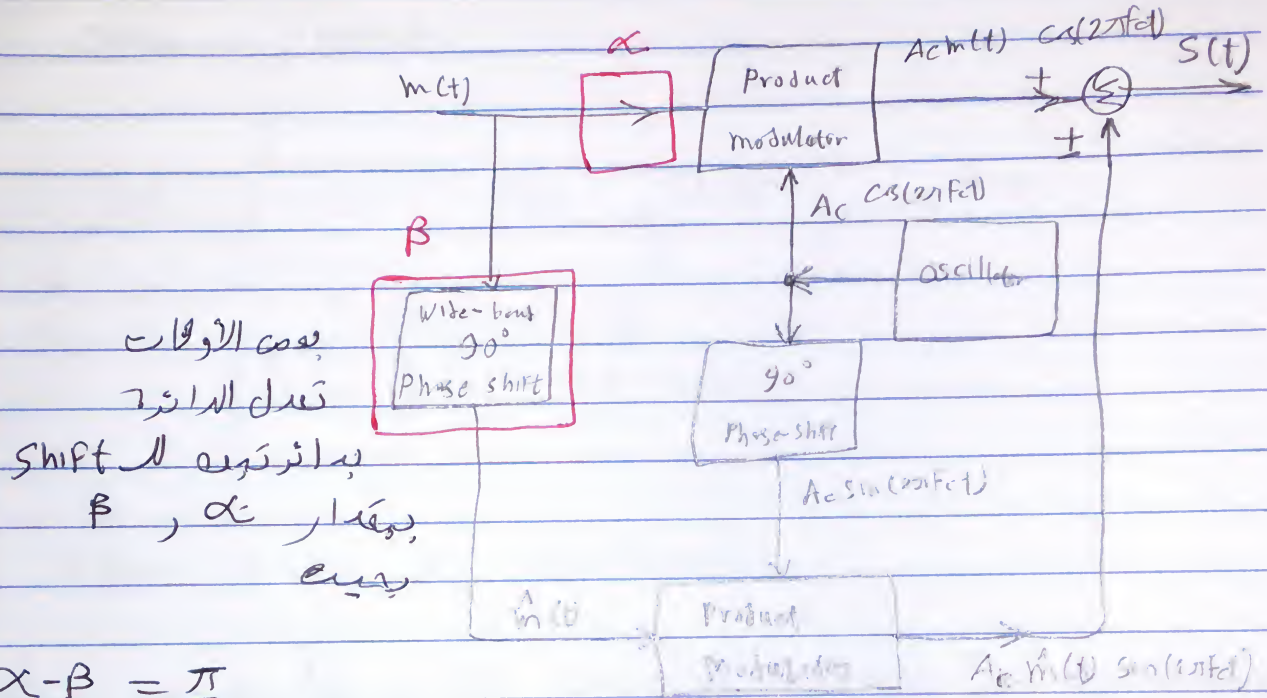
$$S(f) = \frac{A_c A_m}{4} [ \delta(f + f_c + f_m) + \delta(f - f_c - f_m) ]$$





## Generation of SSB Waves :

- ① phase discriminator
- ② Frequency discriminator



$$\alpha - \beta = \frac{\pi}{2}$$

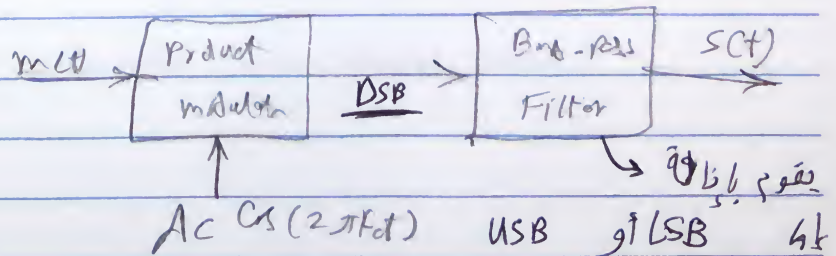
هذا يمكن أن يكون

استقبال ترددات  $m(t)$

ولا يحدد أي إشارات أخرى

Phase discriminator

Frequency discriminator



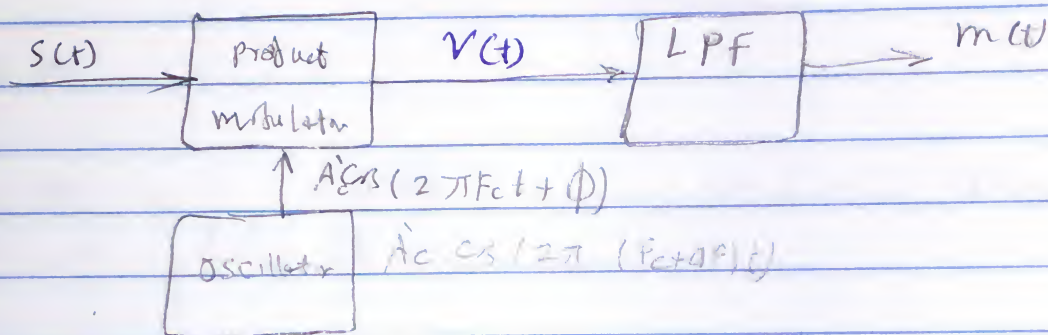
في هذه الحالة لا يمكن استقبال إشارات أخرى



The modulation of SSB :

$$S(t) = \frac{A_c}{2} [m(t) \cos(2\pi f_c t) \mp \hat{m}(t) \sin(2\pi f_c t)]$$

Coherent Detector



$$V(t) = S(t) \cdot A_c' \cos(2\pi f_c t)$$

$$= \frac{A_c A_c'}{2} [m(t) \underbrace{\cos(2\pi f_c t) \cos(2\pi f_c t)}_{\cos^2(2\pi f_c t)} \mp \underbrace{\hat{m}(t) \sin(2\pi f_c t) \cos(2\pi f_c t)}_{\frac{1}{2} \hat{m}(t) \sin(4\pi f_c t)}]$$

$$= \frac{A_c A_c'}{2} \left[ \frac{1}{2} m(t) [1 - \cos(4\pi f_c t)] \right]$$

$$= \frac{A_c A_c'}{4} m(t) \cos \phi \rightarrow \text{Phase error}$$



## Angle Modulation

$$S(t) = A_c \cos(\theta_i(t))$$

Am wave

$$S(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t + \phi) \quad \xrightarrow{\tau \rightarrow 0}$$

$$\theta_i(t)$$

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = \text{instantaneous frequency}$$

FM wave  $\rightarrow f_c$

PM wave  $\rightarrow \phi$

$$S(t) = A_c \cos(2\pi f_c t + \phi_c)$$

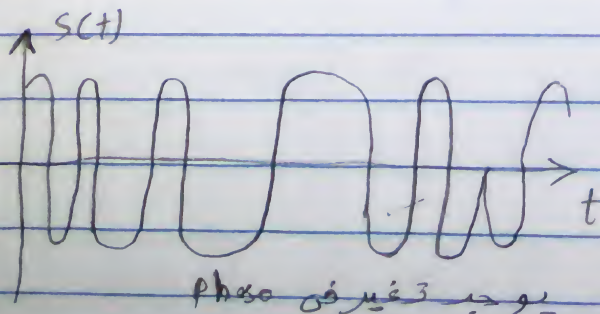
$\underbrace{2\pi f_c t}_{\text{un modulated Frequency}} \quad \underbrace{\phi_c}_{\text{un modulated Angle}}$

$$\theta_i(t) = 2\pi f_c t + \phi_c$$

~~Am~~ PM wave  $\rightarrow \theta_i(t) = 2\pi f_c t + k_p m(t)$

$k_p \rightarrow$  phase sensitivity

$$S(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$





$$f_i(t) = \frac{1}{2\pi} \frac{d\phi_i(t)}{dt}$$

$$\phi_i(t) = 2\pi \int_0^t f_i(t) dt$$

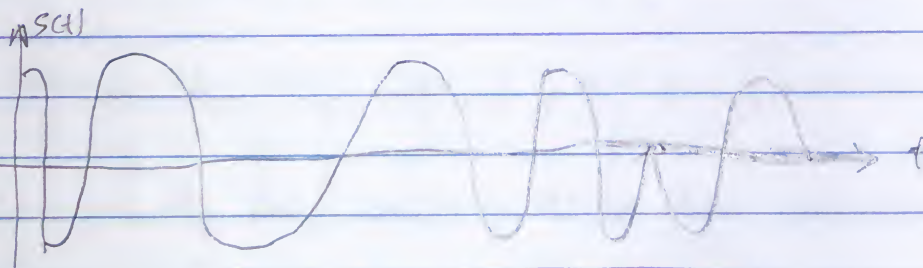
$$FM \rightarrow f_i(t) = f_c + K_f \cdot m(t)$$

$K_f \rightarrow$  frequency sensitivity

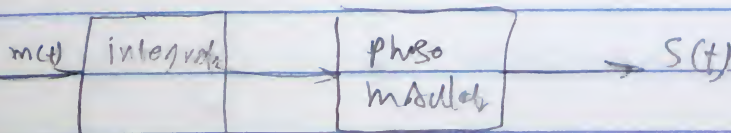
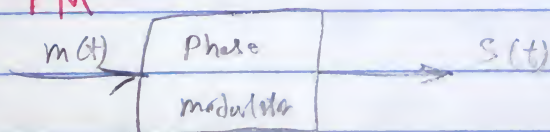
$$\phi_i(t) = 2\pi \int_0^t [f_c + K_f m(t)] dt$$

$$\phi_i(t) = 2\pi f_c t + 2\pi K_f \int_0^t m(t) dt$$

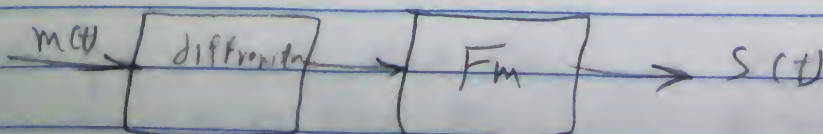
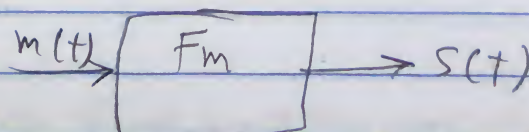
$$S(t) = A_c \cos [2\pi f_c t + 2\pi K_f \int_0^t m(t) dt]$$



FM from PM



PM from FM





FM and PM  $\rightarrow$  better than  $\rightarrow$  AM

نفسه فيها لا تتأثر بـ noise  
لا Amplitude ثابتة

وعنه هو تسمى الدائرة والتكلفة العالية

FM Type :

- ① single tone Fm
- ② multi tone Fm

\* Single tone;

$$m(t) = A_m \cos(2\pi F_m t)$$

$$S(t) = A_c \cos \left[ 2\pi F_c t + 2\pi K_f \int_0^t m(t) dt \right]$$

$$= A_c \cos \left[ 2\pi F_c t + 2\pi K_f \frac{A_m}{2\pi F_m} \sin(2\pi F_m t) \right]$$

$$\Delta F = \pm K_f A_m \quad \text{Frequency deviation}$$

$$S(t) = A_c \cos \left( 2\pi f_c t + \frac{\Delta F}{F_m} \sin(2\pi F_m t) \right)$$

$$\beta = \frac{\Delta F}{F_m} \rightarrow \text{modulation index}$$

$$F_{\text{max}} = F_c \pm K_f A_m = F_c \pm \Delta F$$

$$S(t) = A_c \cos [2\pi f_c t + \beta \sin(2\pi F_m t)]$$

$$\beta \rightarrow \begin{cases} \text{Narrow Band Fm} & \beta < 1 \\ \text{Wide Band Fm} & \beta > 1 \end{cases}$$



Narrow Band FM wave

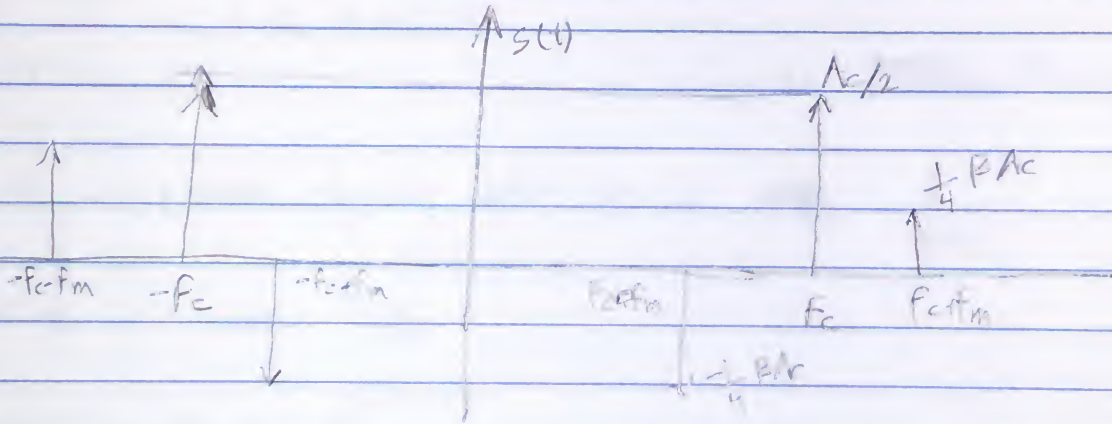
$\beta$  is very small

$$S(t) = A_c \cos(2\pi f_c t) \cos[\beta \sin(2\pi f_m t)]$$

$$- A_c \sin(2\pi f_c t) \sin[\beta \sin(2\pi f_m t)]$$

$$\approx \beta \sin(2\pi f_m t)$$

$$S(t) = A_c \cos(2\pi f_c t) - A_c \beta \sin(2\pi f_c t) \sin(2\pi f_m t)$$



$$S(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} \beta A_c [\cos(2\pi(f_c + f_m)t) - \cos(2\pi(f_c - f_m)t)]$$

$$B.W = 2f_m$$



# \* Wide Band Frequency Modulation

$$\beta \gg 1$$

$$S(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

$$= A_c \operatorname{Re} \left\{ e^{j2\pi f_c t + \beta \sin(2\pi f_m t)} \right\}$$

$$= A_c \operatorname{Re} \left\{ e^{j2\pi f_c t} \cdot e^{j\beta \sin(2\pi f_m t)} \right\}$$

$$\hat{S}(t) = e^{j\beta \sin(2\pi f_m t)}$$

→ Complex ~~Envelope~~ Envelope

$\hat{S}(t)$  → periodic signal with period  ~~$2f_m$~~   $\frac{1}{f_m}$

Using Complex F.S.

$$\hat{S}(t) = \sum C_n e^{j2\pi n f_m t}$$

$$C_n = \frac{1}{f_m} \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \hat{S}(t) \cdot e^{-j2\pi n f_m t} dt$$

$$= \frac{1}{f_m} \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} e^{j\beta \sin(2\pi f_m t)} \cdot e^{-j2\pi n f_m t} dt$$

$$= \frac{1}{f_m} \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} e^{j[\beta \sin(2\pi f_m t) - 2\pi n f_m t]} dt$$

$$X = 2\pi f_m t$$

$$dX = 2\pi f_m dt$$



$$C_n = \frac{f_m}{\pi} \int_{-\pi}^{\pi} \frac{1}{2\pi f_m} e^{j[\beta \sin(x) - nx]} dx$$

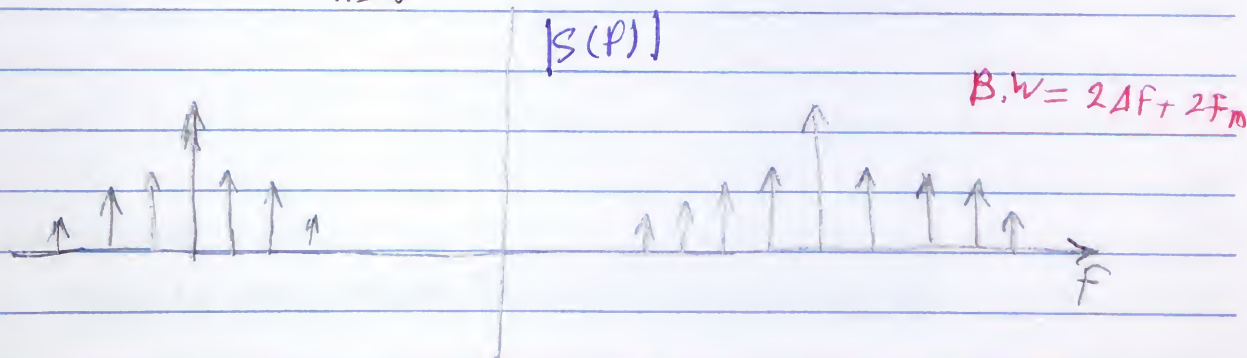
$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j[\beta \sin(x) - nx]} dx \quad \text{Bessel Function}$$

$$C_n = J_n(\beta)$$

$$\hat{s}(t) = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi f_m t}$$

$$S(t) = A_c \operatorname{Re} \left\{ e^{j2\pi f_c t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi f_m t} \right\}$$

$$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi (f_c + n f_m) t)$$



$J_n(\beta)$  properties

$$n=0 \quad J_0(\beta) = 1$$

$$n = \text{even} \quad J_n(\beta) = J_{-n}(\beta)$$

$$n = \text{odd} \quad J_n(\beta) = -J_{-n}(\beta)$$

$$J_n(\beta) = (-1)^n J_{-n}(\beta)$$

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$



## \* Generation of FM waves:

Transmission bandwidth of FM wave,

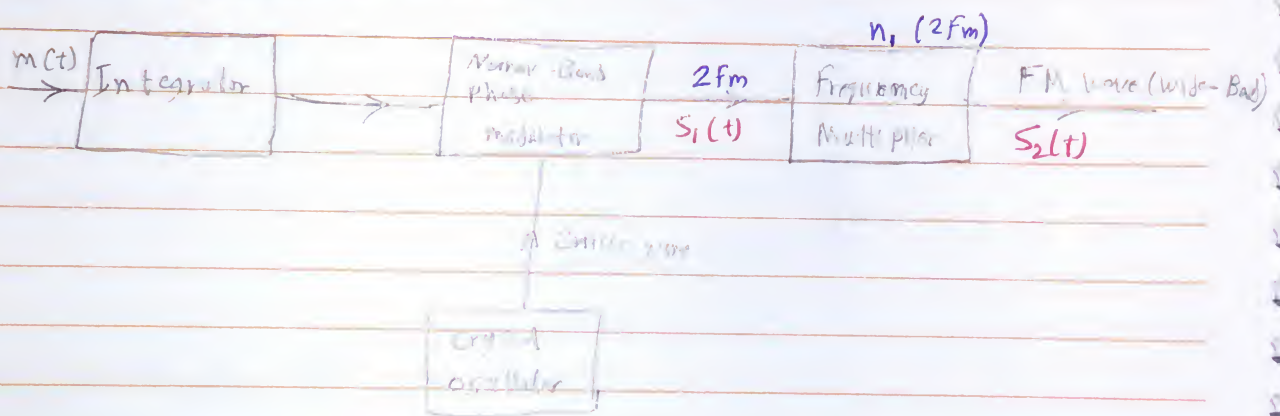
Carson's rule (wide Band FM)

$$B.W. \approx 2\Delta F + 2f_m \approx 2n_{max}f_m$$

$$|J_n(\beta)| > 0.01$$

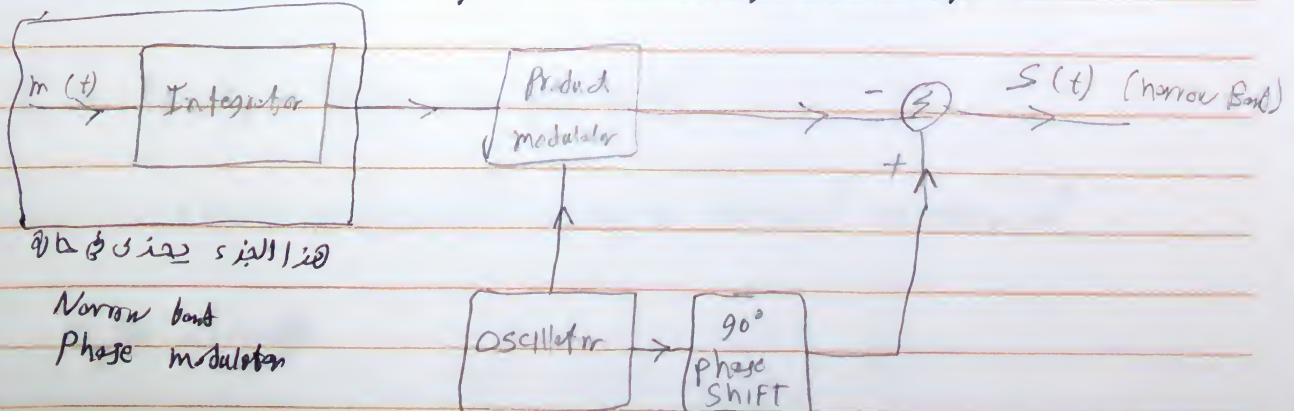
$$\Delta F = \frac{K_f A_m}{f_m}$$

## \* Indirect ~~method~~ FM generator:



Narrow Band (FM)

$$S(t) = A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$



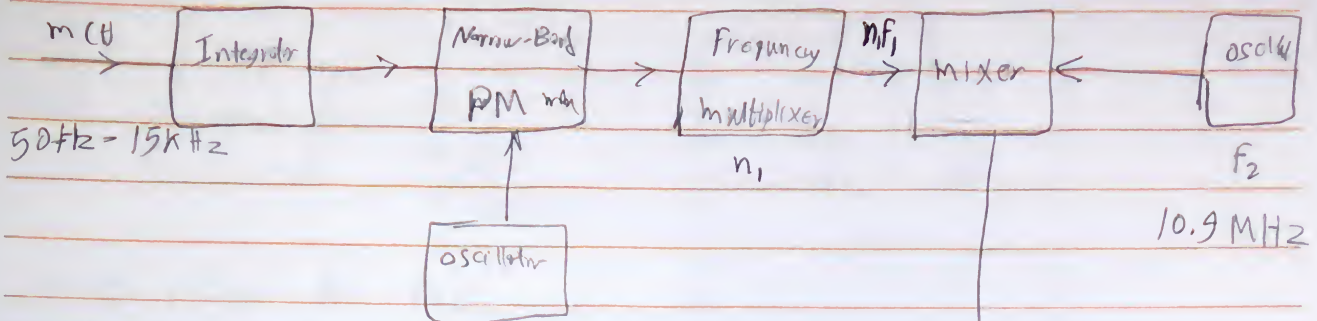
$$S_1(t) = A_1 \cos[2\pi f_c t + 2\pi K_f \int m(t) dt] \quad \text{if } m(t) = A_m \cos(2\pi f_m t)$$

$$S_1(t) = A_1 \cos[2\pi f_c t + \beta_1 \sin(2\pi f_m t)] \quad \beta_1 \ll 1$$

$$S_2(t) = A_1 \cos[2\pi f_c t + n_1 \beta_1 \sin(2\pi f_m t)] \quad n_1 \beta_1 \gg 1$$



Example : Find  $n_1, n_2$



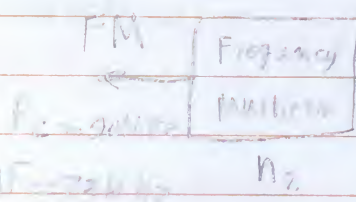
Condition (N.B)

$$\beta < 1$$

$$\beta_{\max} = 0.5$$

$$\Delta F_1 = \beta_1 \cdot F_m$$

$$\Delta F_1 = 0.5 \times 50 \text{ Hz} = 25 \text{ Hz}$$



$$\Delta F_{\text{final}} = n_1 \cdot n_2 \cdot \Delta F_1 = 75 \text{ kHz}$$

$$n_1, n_2 = 3000$$

$$f_c = -n_1 n_2 f_1 + n_2 f_2 = 90 \times 10^6$$

$$-3000 \times 0.2 \times 10^6 + n_2 \times 10.9 \times 10^6 = 90 \times 10^6$$

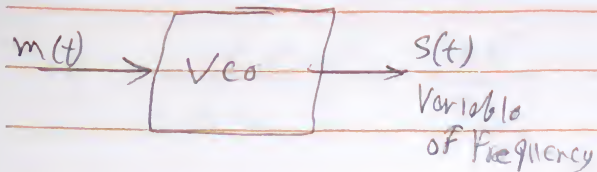
$$n_2 \approx 63$$

$$n_1 \approx 48$$



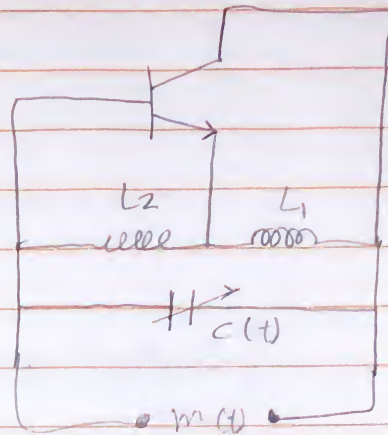
\* Direct methods Direct FM generator

## \* Voltage Controlled Oscillator



$$C(t) = C_0 + \Delta C m(t)$$

$\downarrow$   
 Capacitance in the absence of modulation  
 $\downarrow$   
 base band signal



Hartley oscillator  
VCO

$\Delta C \rightarrow$  max change of capacitance

It is a variable capacitor

P-n Junction with reverse bias  
 $\rightarrow$  Varactor or Varicap

$$f_i = \frac{1}{2\pi \sqrt{(L_1 + L_2) C(t)}} = \frac{1}{2\pi \sqrt{(L_1 + L_2) [C_0 + \Delta C m(t)]}}$$

if  $m(t) = \cos(2\pi f_m t)$

~~$$f_i = \frac{1}{2\pi \sqrt{(L_1 + L_2) C(t)}}$$~~

$$f_i(t) = f_0 \left[ 1 + \frac{\Delta C}{C_0} \cos(2\pi f_m t) \right]^{-1/2}$$

$$f_0 = \frac{1}{2\pi \sqrt{(L_1 + L_2) C_0}}$$



$$F_1(t) = F_0 \left[ 1 + \frac{\Delta C}{C_0} \cos(2\pi f_m t) + \dots \right]$$

$$\boxed{\Delta C < C_0}$$

$$\frac{-\Delta C}{2C_0} = \frac{\Delta F}{F_m}$$

$$\therefore F_1(t) = F_0 \left[ 1 + \frac{\Delta F}{F_m} \cos(2\pi f_m t) \right]$$

Instantaneous Frequency of FM wave

Disadvantage  $\rightarrow$  \* Un stable Frequency

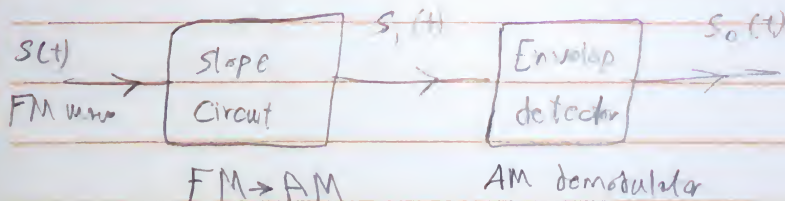
\* Variable Capacitance is not Linear

\* Detection of FM wave:

① Slope detector

② PLL  $\rightarrow$  Phase Locked - Loop

① Slope detector



Slope circuit  $\rightarrow$  differentiator

$$S(t) = A_c \cos \left[ 2\pi f_c t + 2\pi K_f \int_0^t m(t) dt \right]$$

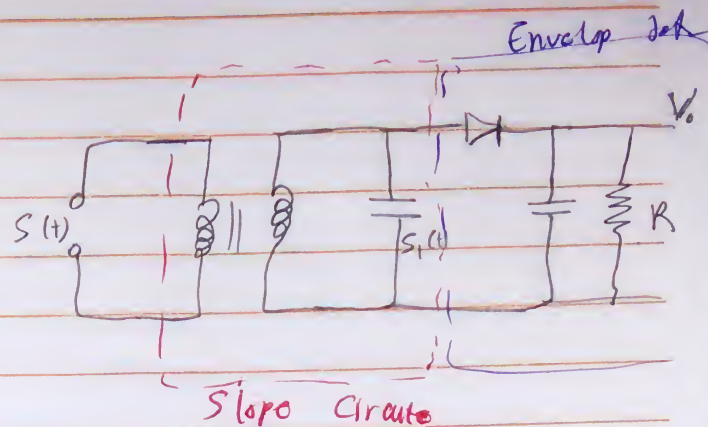
$$S_1(t) = -A_c \left[ 2\pi f_c + 2\pi K_f m(t) \right] \sin \left[ 2\pi f_c t + 2\pi K_f \int_0^t m(t) dt \right]$$

$$S_0(t) = -A_c \left[ 2\pi f_c + 2\pi K_f m(t) \right]$$

$$S_0(t) \propto m(t)$$



Slope Circuits



② phase-locked-loop:

